The **ScienceMath** Project

Mathematical Literacy And Cross-Curricular-Competencies
Through
Interdisciplinarity, Mathematising and Modelling Science

An European Cooperation Project
Between Universities and Schools
Of Germany, Denmark, Slovenia and Finland,
Supported by the European Commission
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Results and Material

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Content

Preface

This volume documents the European ScienceMath project. It summarises its background, work in the project and results. It relates to the development and research activities and includes interdisciplinary teaching modules between maths and science. Its development was the main objective of the project.

This volume should give an overview as well as a quick insight into the different project areas. Therefore the chapters are short but touch the main key-words. This should motivate a deeper occupation with the project. More information and more detailed descriptions, more teaching material and background presentations for direct download are available at the project´s website www.sciencemath.ph-gmuend.de and in the referred literature.

The ScienceMath Team, autumn 2009

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1 The Project ScienceMath and its Background

1.1 Short Description

The project ScienceMath is an interdisciplinary European co-operation project between mathematics and sciences. It is supported by the European Commission within programme Comenius 2.1. Co-operation partners are universities and schools of Germany, Denmark, Finland and Slovenia. Objective is the development of proven teaching sequences and –modules that lead to a comprehensive and multidimensional learning of mathematic contents and concepts. The basic idea is to encourage mathematic learning in scientific contexts and activities of the pupils. This is in direct accordance with the OECD requirements. Here the project offers concrete teaching and learning material to integrate the formal aspects of the European mathematical lessons into interdisciplinary and applicable contents for the promotion of mathematical literacy.

The basis of the project is an interdisciplinary approach with sciences especially with Physics. The pupils shall experience Mathematics in an appropriate interesting and important way by the means of extra-mathematical references; learning in interrelations shall contribute to an intuitive mathematic understanding. With the aid of scientific contexts and methods the gap between formal mathematics and authentic experience shall be closed and on the other hand the variety of mathematic items shall be experienced.

The theoretic background for this approach is the Realistic Mathematics Education concept that has been developed at the Freudenthal Institute in the Netherlands. According to this context related problems are central parts of mathematics which may not only be applications but which shall also lead to a direct debate about mathematic contents and concept in the context.

Problems to support e.g. the learning of the concept of function shall be posed in a way that pupils are motivated to comprehend the basic process of a functional context in a real, authentic and experimental situation.

1.2 The ScienceMath Approach: "Math through Science"

Theoretical background of the ScienceMath Approach is the idea of supporting mathematical learning through physical, biological etc. contents and methods. Sciences offer the possibility for realistic teaching. Concrete physical or biological correlations may initiate mathematic activities and lead to authentic experiences. Mathematic contents and methods are apprehended in reasonable contexts; reality of pupils may be extended by mathematical understanding. Various realistic references lead to different models and may so contribute to distinction of conceptual attributes and of different models. The variety of scientific phenomena allows open terms of references and so self-dependent development of mathematics. Mathematic items, e.g. the concept of function, may be experienced as modelling tools. The coherences of meanings and the differing attributes may be detected within various realistic references.

Scientific experiments are an encouraging approach in interdisciplinary learning of mathematics. The instruction may consist of the following: collecting of data out of the experiment, development and investigation of a mathematic model in different representation forms and reflection on the mathematic content of the model. Taking experiments as basis the pupils will have to chose variables and representation forms and set them into correlation. The extension to scientific connections creates a setting for the pupils that challenges justified analyses, authentic experiences and variation between different representation forms. The table e. g. is first of all a model of data collection and becomes later a model for the discussion about the functional relation between the recorded sizes.

A specific advantage of experiments relates to the learning of mathematic concepts. In connection e.g. to the concept of function the experimental steps correspond to the different stages of learning the concept: If a running car is watched while the covered distance is measured at certain points of time the aspect of mapping may be realized. The aspect of covariation, here: the concurrent change of distance and time, can be experienced by watching the stopwatch while the car is going along a staff.

1.3 Theoretical Background

The main project´s aim is the development of tested teaching modules resp. sequences according to the ScienceMath Approach. This includes the aim of supporting mathematical literacy and authentic experiences through interdisciplinary/ cross-curricular teaching.

The ScienceMath modules are evaluated from different perspectives. One evaluation line focuses on the teaching activities with experiences and systematic observations/research. Another line stresses the theoretical background and the development of evaluation methods. During the project run a description of the main background concepts could be managed. This took place on the backdrop of international research results and leaded to an easy module evaluation through operationalising (see 2.2.3).

1.3.1 Mathematical Literacy[1]

The international studies about students´ mathematical competencies focus on the term *mathematical literacy* as the agreed aim of mathematics education.

> *"Mathematical literacy is an individual's capacity to identify and understand the role that mathematics play in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the need of that individual's life as a constructive, concerned and reflective citizen."* (OECD 2006, p. 72)

The conception emphasises the application of mathematics in authentic situations; the process of *mathematisation* is central. Students shall structure a problem, translate it into formal mathematical language, solve it within mathematics and interpret it with respect to the initial problem. For structuring, solving and interpreting problems mathematical competencies are necessary. The OECD lists eight competencies: *thinking and reasoning*, *argumentation*, *communication*, *modelling*, *problem posing and solving*, *representation*, *using symbolic, formal and technical language* and *operations and use of aids and tools*.

The PISA framework was made to test students' performance in mathematics. Besides mathematisation, no concrete approaches for teaching mathematics as a con-

[1] This chapter summarises the comprehensive exposition in (Zell 2009). For more detailed argumentation see the original reference.

tribution to mathematical literacy can be derived. Schoenfeld (Schoenfeld 2001) calls for "quantitative literacy" which means "the predilection and ability to make sense of various modes of mathematical thought and knowledge to make sense of situations we encounter as we make our way through the world." It includes confidence with mathematics; a cultural appreciation of mathematics; the ability to interpret data, to think logically, to make decisions thoughtfully, to make use of mathematics in context; and more. He notes that "the mathematical skills that will enhance the preparation of those who aspire to careers in mathematics are the very same skills that will help people become informed and flexible citizens, workers, and consumers."

The most explicit definition of general aims in mathematics is done in (Kilpatrick et al. 2001). The authors analysed, how successful learning of mathematics can be characterised and should give hints to teachers and researchers. As a definition/synonym they chose the term "mathematical proficiency" which fitted the best, in their opinion. Mathematical proficiency contains five strands, which are interwoven and correlate to each other. By solving mathematical problems all of these strands can be touched. They are

> *conceptual understanding* (comprehension of mathematical concepts, operations, and relations), *procedural fluency* (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), *strategic competence* (ability to formulate, represent, and solve mathematical problems), *adaptive reasoning* (capacity for logical thought, reflection, explanation, and justification) and *productive disposition* (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy) (Kilpatrick et al. 2001).

According to (Winter 1995) math lessons should convey three interdependent fundamental experiences to the students. First they should perceive and understand phenomena, which (should) concern us all in a specific way. Mathematical items shall be experienced as a deductive theory and at last they should learn heuristic competencies which can be applied outside of mathematics, too.

In almost every paper applying mathematics and solving problems is mentioned. To solve problems *heuristic thinking* is needed. Heuristic thinking is not very strict but plausible thinking to achieve a solution of a given problem (Polya 1949). It is a typical thinking during a problem solving process. A centre of a conception of mathematical literacy must be heuristic competencies which allow structured approaches with plausible steps. These approaches should be applicable for inner and outer mathematical contexts. Heuristic thinking is more than schematic application of mathematics since people are aware what they are doing. It fits very well to a conception of mathematical literacy since it stays on a level of plausible and logic reasoning and doesn't contain strict mathematical proofs. To characterize heuristic thinking a list of heuristic competencies follows. They describe heuristic procedures and the correspondent thinking processes. They are independent of any mathematical content and may overlap if solving problems. Attributes of **heuristic thinking** are

- *Comprehend information, problem and essential components* which include heuristic strategies *inductive reasoning*, *analogies, specialisation, decomposition and combination, sketches, identifying essential components*[2]

[2]These strategies needn't be applied on a formal level. If applied on a descriptive level and results are adequate to a given problem, this will be satisfactory.

- *Changing within inner mathematical representations*
- *communicate*
- *Reflecting and interpreting steps of argumentation and result*
- *sensible use of aids and tools.*

Applying problems with mathematical contents require *comprehensive understanding of mathematical concepts and procedures*. If one wants to argument plausibly he needs to know both meaning of concept or procedure itself and their functional meaning. Mathematical concepts and procedures should therefore be taught in an *integrated and functional way*. This follows the definition of conceptual understanding in (Kilpatrik et al. 2001). Implicitly this definition also includes teaching concepts *multifacetedly*. Then students are more likely to recognize, why a mathematical idea is important and in which contexts it can be applied. From a pedagogical and sociological view it seems to be better to make conceptual groups than to list containing all mathematical concepts to be mathematically literate. Last would however be given to covering concepts and not to *understanding* them; on the other hand the needs of a country or culture may differ (Jablonka 2003). The following classification is according to (DOE 2003): Numbers and Operations in context, Functional relationships, Space, Shape and Measurement and Data handling. This classification appears useful, since it gives a conceptual framework and doesn't go into detail. The concepts and procedures needed depend on type of school and the aims of a community.

The last aspect of this conceptualisation of mathematical literacy is *familiarity with deductive reasoning.* Math is a strong deductive theory. That's why mathematically literate persons should know the main features of this theory and be familiar in deductive reasoning. Argumentation with knowledge already known is equivalent to deductive reasoning in whatever context. At the latest when contents become abstract and intuitions won't help anymore, one needs logical reasoning based on deduction.

Supporting mathematical literacy is a stressed aim in the ScienceMath project. This requires e.g. a definition of mathematical literacy which allows a respective evaluation of the developed modules. The above given information is seen as a good basis for a comprehensive definition of mathematical literacy, which allows an evaluation of modules and lessons as well (see 3.1.3).

Summarising (Zell 2009, trans.):

Mathematical literacy can be characterized by three aspects:

1. Heuristic thinking which allows structured and plausible approaches that can be applied to inner and outer mathematical problems.
2. comprehensive understanding of mathematical concepts and procedures within the conceptual groups of
    - Numbers and Operations in context
    - Functional relationships
    - Space, Shape and Measurement
    - Data handling
3. familiarity with deductive reasoning.

1.3.2 Scientific Literacy

The main objective of the ScienceMath project is the development of teaching modules that support the learning of mathematics and contribute to *mathematical* literacy (1.3.1). Background idea of the ScienceMath Approach is learning mathematics through science (1.2). I.e. that all the modules contain science, which is more or less central. Against this background they should contribute to *scientific* literacy as well.

Scientific literacy according to PISA differentiates three main aspects: knowledge of science, which refers to knowledge about the natural world; knowledge about science, which refers to the way science works and attitudes toward science. Scientific literacy emphasises applying scientific knowledge using cognitive processes which are typical to scientific enquiry to "identify questions, acquire new knowledge, explain scientific phenomena and draw evidence-based conclusions about science-related issues" (PISA). Knowledge of science covers phenomena of physical, living, technology and earth and space systems. The respective ScienceMath modules concern these aspects (see 4). Knowledge about science means knowing how science work. Two categories are distinguished: scientific enquiry and scientific explanations. The way how scientific knowledge is acquired is covered by scientific enquiry; scientific explanations refer to the way data is interpreted and theory is created/referred. Respective ScienceMath modules touch these aspects or could be expanded in that way.

Major aspects in the conception of scientific literacy are: identifying scientific issues, explaining phenomena scientifically and using scientific evidence. Identifying scientific issues means comprehending scientific information on given topics; explaining phenomena scientifically is very much related to the aspect of scientific explanations, which is covered in knowledge about science. And using scientific evidence includes making sense of scientific findings as evidence for claims and conclusions.

The scientific competencies imply cognitive processes, namely inductive/deductive reasoning, critical and integrated thinking, transforming representations, constructing and communicating arguments and explanations based on data, thinking in terms of models and using mathematics. Scientifc literacy is also seen as a continuum from less developed to more developed scientific literacy like mathematical literacy.

1.3.3 Authentic Experiences

We already mentioned that it is an important aim of the ScienceMath project to help to close the gap between formal mathematics and authentic experiences (1.1). This should be reached by the use of methods and contents of science. According to PISA authentic contexts mean situations, in which mathematics is used and genuinely direct to a problem to be solved. This is in contrasts to situations, which are made up to practice mathematics. For the testing four different contexts are differentiated: personal, educational/occupational, public and scientific. It should be noted that authentic doesn´t intend to indicate that the items involved in a situation are in some sense genuine and real.

1.3.4 Interdisciplinarity

The ScienceMath Approach is an interdisciplinary approach between mathematics and sciences like physics, but also biology, chemistry and geography. Thus realisation in school needs knowledge about interdisciplinarity. In the ScienceMath group there is a wide range of competencies: There are researchers who worked on the theoretical background and realisation concepts in practice as well as experienced teachers from different subjects who cooperate interdisciplinary. E.g. the mathematics teachers Rugelj from Slovenia and Youd from Finland can look back on deep experiences while trying out the ScienceMath material in close cooperation with the science teachers Golež from Slovenia and Päiivi from Finland. Furthermore there are long-lasting experiences in interdisciplinarity in in-service teacher education (Merenluoto, Beckmann, Michelsen and Kobal). Further important work of the partners concern the development of a conceptional framework for cross-curricular/ interdisciplinary teaching (Beckmann from Germany) and a training concept for implementation interdisciplinarity in the classroom (Michelsen from Denmark). This will be described in 2.

1.4 Areas of Development

1.4.1 Introduction

The modules developed and tested in the ScienceMath project focus especially on

- Mathematics through every-day-life problems and scientific questions,
- Modelling Processes and Integrating Mathematics and Sciences,
- Learning concepts like function and variable through experiments.

1.4.2 Mathematics through Every-Day-Life Problems and Scientific Questions

Relevant modelling activities of real life questions may lead to all mathematic fields. Within the ScienceMath project interdisciplinary teaching sequences resp. modules concerning different mathematic contexts are developed and tested. Real life impulses concern the universe, nutrition, sound, digital, earthquake, satellite dish, traffic and car motion, light refraction, insurance premia and many many more. The teaching modules consider e.g. experiments to the concept of function and variable, calculus through real-time-measurements, an integrating sequence concerning the physical phenomenon of refraction etc. (see 4).

The Module “Parabola between Mathematics and Physics - The Case of the Horizontal Launch” (Golež) – for example – starts with a real life-problem and a scientific investigation. It is impulsed by the question of the so called “Two coin problem”. That is the question of which of two moving coins will be first on the floor: The coin which will fall down as a freely falling object or that which will be launched in horizontal direction. After a few repetitions of an adequate experiment, one will agree, that you can hear only one “clap”, i.e. the two coins are falling downwards with equal speed. This leads to the conclusion: movement of a horizontally thrown object can be split into two. An enhanced experiment enables to measure quantities which lead to a description of the horizontal part of this movement: A ball is shot by a spring gun and the movement is analysed through a projection onto a white board (figure 1).

In continuing the teaching module the results of the physical experiments and the physical discussion motivates an interesting mathematical investigation, which could take place in different mathematical lessons and relate to concepts from functions (linear, quadratic) to calculus (e.g. derivative). Mathematics is experienced adequate and in a meaningful manner.

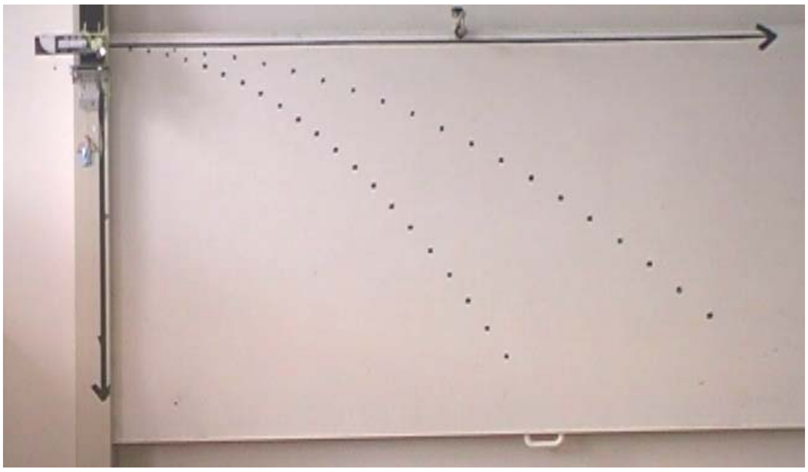

Figure 1
Whiteboard projections of horizontal launch

1.4.3 Modelling Processes and Integrating Mathematics and Sciences

A central approach in research and development of ScienceMath is the idea of "expanding the domain" (Michelsen 2001) – that is expanding mathematics by integrating e.g. biological and physical procedures, phenomena and contents. By means of interdisciplinary contexts pupils shall develop competencies through mathematising and modelling activities. Integrated modelling courses are developed and tested.

In the module "Modelling Things in Traffic" – for example – the question "How long is the duration of the yellow light?" is discussed and answered along different modelling activities (Michelsen & Nielsen, figure 2): The activities start with a verbal model: the students approach to the question through their own experiences and identify important developing factors like reaction time of the driver or stopping distance of the brakes. A real model of the situation is build and discussed with means of a physical model (e.g. force vectors). This leads to a (simplified) mathematical model, which allows calculations and predictions. The integration of reality, science and mathematics concludes in concrete measurements at a crossroad and the evaluation of the own model. Mathematics is learned adequate and meaningful.

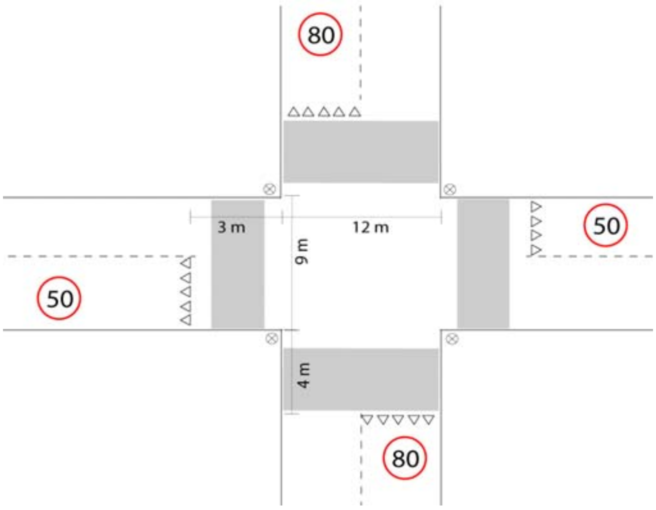

Figure 2
Model of a cross-road

1.4.4 Learning concepts like function and variable through experiments

One line of the ScienceMath project follows up the approach of supporting the learning of the concept of function and variable through experimental activities (Beckmann, Päivi, Zell and more). For this experiments have been developed for different functional contexts and have been tested in various school forms of lower and early higher secondary school. The experiments are composed in a way that the requested material, the structure and the performance are simple and uncomplicated and can therefore be introduced within every day's course in mathematics. The experimental and textual work will be stimulated by worksheets which start always with an impulse from every day life. The experiments can be introduced in a single experiment e. g. for experimental investigation of certain functional contexts (e. g. proportional, quadratic, inverse proportional etc.) or as stage within stage learning in order to elaborate selected functional contexts through commonalities or differences. The experiments however may serve for experience and investigation of functional dependencies and support of functional thinking (in stages with different contexts).

The module series "functional relations" 1, 2 and 3 – for example – support the learning of the aspects of functions: the correspondence aspect (action layer) is experienced through the acquisition of data. While dipping in a ball with a special radius into a water jug a corresponding water volume is displaced (figure 3). The covariation aspect (process layer) is experienced through experimental observations: While the time goes by the distance a car goes arises continuously (figure 4).

Figure 3
Correspondence between radius of the ball and displaced water

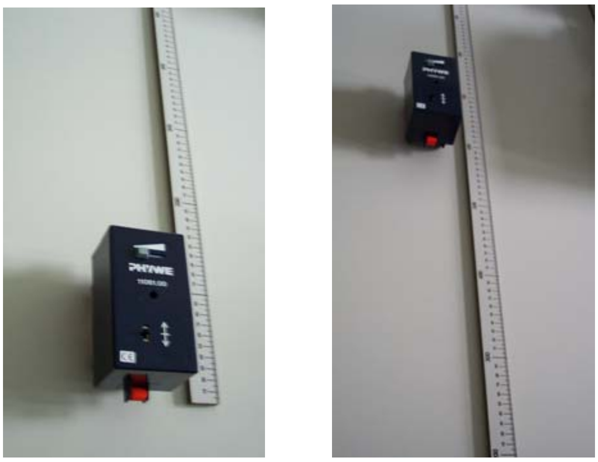

Figure 4
Covariation of time and distance

2 Interdisciplinarity/ Cross-Curricular Teaching

2.1 About Interdisciplinary Teaching

2.1.1 A Model

This chapter summarises theoretical background ideas about interdisciplinary teaching according to the model developed by Beckmann. The model includes different forms of cooperation, which is seen to be noteworthy – especially for implementation in the classroom. E.g. it offers the chance to start interdisciplinary

teaching firstly on a lower level of cooperation. The whole conceptional framework for cross-curricular teaching is available as a special issue of the journal "The Montana Mathematics Enthuisiast" at[3]
www.math.umt.edu/TMME/vol6supp/TMMEvol6_supplement1_March2009.pdf

A definition, first part:

> *Cross-curricular/subject-integrative instruction* means dealing with a (subject-related or non-related) topic, in which the subject borders are exceeded and other subjects are integrated.
>
> The instruction is done in *co-operation.*
>
> The co-operation can consist of topic- and major subject-related work (TM-Form, level 1), of parallel topic related work (PT-Form, level 2), of parallel planning work (PP-Form, level 3) and of the planning of joint work (JP-Form, level 4).
>
> It may be example oriented, course oriented or project oriented.
>
> Instructions on levels 1 and 2 is termed *cross-curricular*, on levels 3 and 4 it is termed *subject-integrative.*

Explanation:

*Cross-curricular teaching***:**

TM-Form (Topic- and Major Subject –Related Form):

The co-operation starts with the teacher´s realisation that the teaching needs to go beyond the subject/disciplinary boundaries and that the contact and methods of other subjects need to be used. The colleagues from other departments are involved by providing complementary aspects that consolidate the major or central subject. The area of planning only extends to certain elements of the other subject or subjects (figure 5). We speak of a special case if the teacher, who teaches in a cross-curricular way, has a multiple competence.

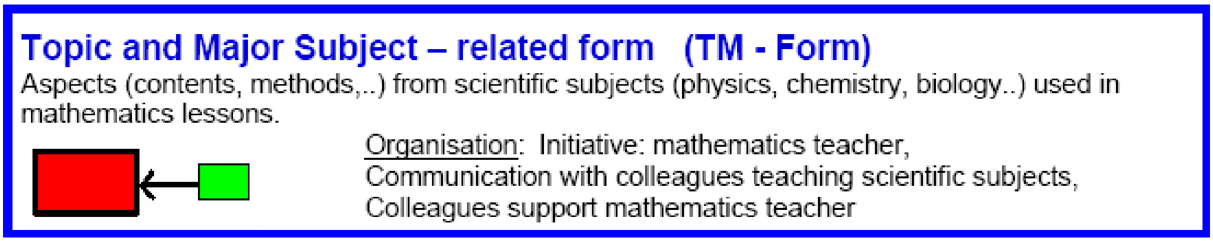

Figure 5
TM-Form, short characterisation

PT-Form (Parallel Topic-related Form)

Here, several colleagues participate in the group work from the very beginning. The starting points are content that is common to various subjects. The teachers co-ordinate their teaching contents for the whole year aiming at teaching the common content, if at all possible, at the same time. The emphasis of the co-operation during the school year is in particular on an intensive exchange of ideas and temporal agreements (figure 6).

[3] information 16th October 2009, (Beckmann 2009), see also German edition: Beckmann 2003

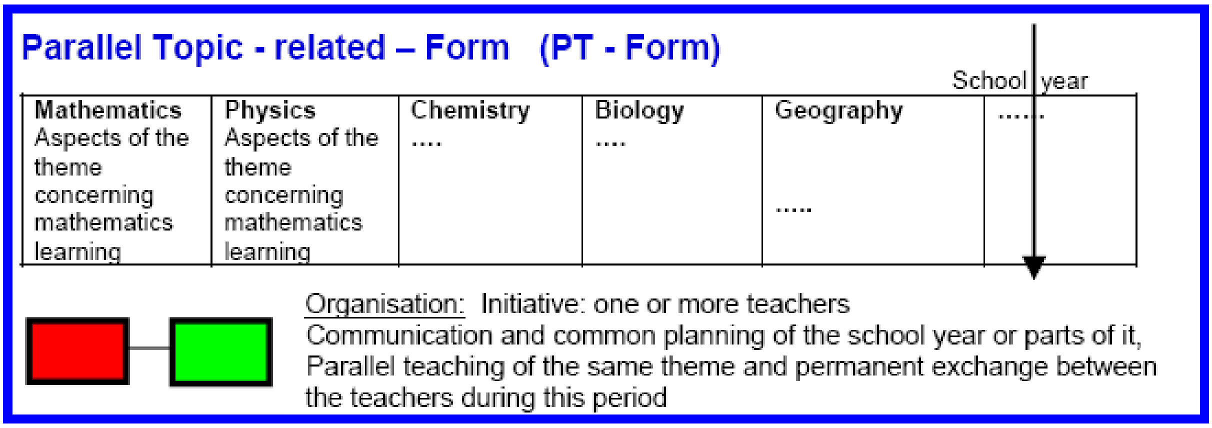

Figure 6
PT-Form, short characterisation

Subject-integrative Teaching

PP-Form (Parallel Planning Form)

The collaboration of a team of teachers is motivated by a teaching topic that can/has to be dealt with jointly in many subjects. Here, the teachers plan the units jointly; they are in constant contact with each other exchanging ideas. It is also possible the coordination and her subject then becomes the major one. The intensive exchanges during the planning relate to the content, the methods and the objectives, the competencies to be acquired by the students but also the implementation of the organisation.

In the planning, at least one adjournment or exchange of subjects must be taken into consideration, because it may become necessary in the course of the parallel work that certain subject areas have to be finished first, before the content of another subject can be worked on (figure 7).

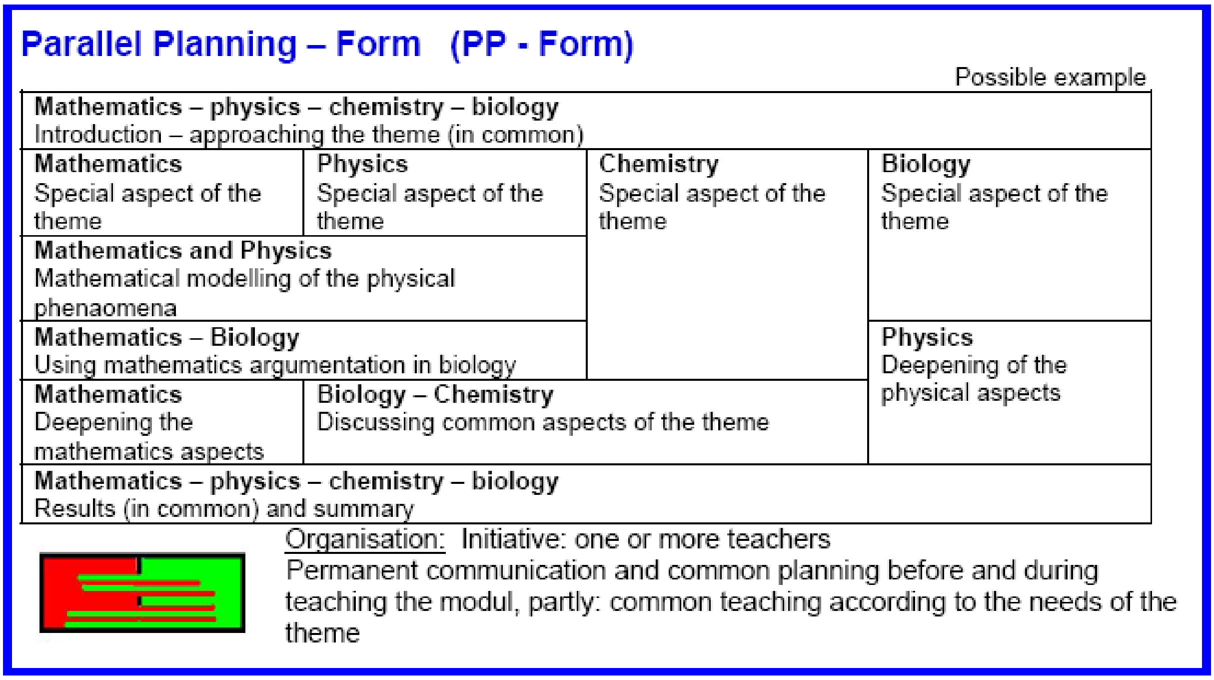

Figure 7
PP-Form, short characterisation

JP-Form (Joint Planning Form)

At the highest level of co-operation, the planning is done so closely that the complete instruction is done in group work. Planned group work is the most complex form of co-operation and an essential extension of the forms of co-operation. The framework for the instruction is a topic or topic area that can only be mastered comprehensively in collaboration with several subjects. Typically, there are longer phases, in which some of the subjects involved do not play a clear role. On the other hand, however, all subjects are constantly and equally involved in the planning, so that, in principal, the special case of a co-ordination through a major subject is irrelevant (figure 8)

Joint Planning – Form (JP - Form)

Organisation: Initiative: one or more teachers
Team teaching: **All subject melt together to one subject!**
Possible: project-oriented teaching with subject-oriented project parts.

Figure 8
JP-Form, short characterisation

A definition, second part

Exceeding subject borders results in *contact* with other subjects. Here, common interest (*Idiosyncratic Aspects*) but also alien interests (*Alien-ness*) meet. They may be related by making use of the alien aspects, by integrating the alien aspects or by mixing subject and alien aspects.

Explanation:

On the Term Idiosyncratic Aspects (IA)

Starting from a certain subject, the term idiosyncratic aspect denotes the subject areas (content, methods, objectives etc.) which are validated by the subject´s didactic principles. These may be general aspects, such as the deductive method, but also singular aspects like doing classical geometrical constructions with straightedge and compass.

On the Term Alien Aspects (AA)

Starting again from a particular subject, the term alien aspect refers to the idiosyncratic aspects of a different subject (co-operating subject), which differ from the aspects of the original subject. The differences may be total, e.g. literature as subject material, or may only concern parts of a common aspect. Alien aspects can thus be recognised by comparing common aspects. In a co-operation, the idiosyncratic aspects of the individual subjects become alien aspects as far as they distinguish themselves from those of the co-operating subject.

In the lessons alien-aspects can be used, integrated or mixed with idiosyncratic aspects. An example for using aspects is the use of experiments in math teaching. Examples for integration are broaching the subject of common forms of presentation in chemistry and maths, the description of a tune through vectors in music or learning the construction of pie charts through the nutrition circle. By mixing the aspects the alien aspects do not just clash, but they have to be put in relation to each other.

A definition, third part:

The interest in cross-curricular/subject-integrative teaching lies in an *enrichment* of the subjects. Here, the interest can orient itself to common aspects or to alien aspects. It may be oriented to content, methods, competences and ways of thinking, and it may appear in the form of subject orientation, parallel orientation and comprehensive orientation.

2.1.2 Research results about Realising Interdisciplinarity

In spring 2009 questionnaires were sent to 180 schools in Baden Württemberg (federal state in Southern Germany) to ask about their views on interdisciplinary teaching and its realisation in school. During the same period 10 teachers from 6 different schools were interviewed (Zell 2010). 184 teachers from 70 different schools have answered the questionnaire. Most of the teachers who participated are in favour of interdisciplinary teaching. The majority believes that interdisciplinary teaching is a good way to look at a theme from different views, helps students to a deeper understanding of mathematical concept and procedures, which can better remembered. Although they appreciate interdisciplinary teaching, they teach this way only occasionally. If taught interdisciplinary they teach by themselves most of time. A small minority cooperates with colleagues from other subjects. They would like to teach it more often, especially they would like to coordinate interdisciplinary themes with colleagues and prepare lessons together if a topic is suitable.

Considering the realisation of interdisciplinary teaching there are major problems in time, cooperation and education in interdisciplinary teaching. According to the teachers involved in this survey, most of them said that preparation was time consuming and they didn't have enough time for that. Daily routine in schools makes it difficult for cooperation in school. A big majority thinks that their university education in interdisciplinary teaching was not appropriate.

Summarising, teachers would like to be helped by their effort in cross-curricular teaching. This concerns teacher education as well as the daily routine in school. Offering prepared interdisciplinary teaching material for the direct use as done in the ScienceMath-Project is one way to promote interdisciplinary teaching. It shows possibilities to teach interdisciplinary, saves time for preparation und may initiate cooperation among teachers (see also 2.2).

2.1.3 Experiences from the Project

According to the different forms of interdisciplinary cooperation different kinds of modules were developed and tested. We learned that – for a successful cooperation - the relation between the cooperating teachers is a very important basis. There has to be an acknowledgement about knowledge and competencies of each other. There should be an understanding and agreement about contributing with own aspects and overtaking ideas from each other. On the other hand the cooperating person has to feel like a real cooperating person and has to be aware with the module. E.g. in one of the teaching trials the biological teacher was only waiting for her entry and didn´t take own initiative.

For all cooperating teachers interdisciplinarity was a challenge, which leaded to new perspectives. They stressed the value of the scientific part of the mathematical lessons. Some mentioned initial difficulties to accept and to integrate a different view on a topic; but looking back they felt enriched. Some teachers were afraid of the time aspect, but they learned that the ScienceMath-module should not be used additional, but instead of traditional lessons. Teachers pointed out the long-term effects and re-usability at many different mathematical themes during the whole schooldays.

We did not feel that there were differences between the countries in accepting interdisciplinary lessons; more the differences concern the individuality of the teacher. It could be noticed – perhaps - that there is a longer tradition in open oriented and interdisciplinary mathematical lessons in Scandinavia than in Eastern European countries; but summarising we observed the motivating impulse of the ScienceMath- Approach for all.

Last but not least it is a very important result of the ScienceMath project, that the developed modules motivated teachers to try them out and to get into cooperation and integrate interdisciplinarity in the classroom. This concerns European teachers who visited the ScienceMath-website as well as teachers who participated at teacher trainings. It promises a continuing European exchange in future.

2.2 Implementation of Interdisciplinarity in the Classroom

2.2.1 Introduction

The advantages of interdisciplinary lessons can only develop by realising it in the classroom. Central basis for an easy and successful implementation are prepared teaching modules and an implementation concept. The ScienceMath project offers both.

2.2.2 The ScienceMath – Professional Development Concept

Background of the ScienceMath-Professional Development Concept for interdisciplinary mathematical lessons resp. the framework for a teacher training concept is the positive evaluated ScienceMath-European teacher training event in Slovenia 2009 (local organisors: University of Ljubljana and St. Stanislav Institution Ljubljana, Kobal, Golež & Rugelj). The concept in its whole relates also to ideas of the modelling courses for secondary in-service teachers in the Master-programme of the University of Southern Denmark (Michelsen 2008).

The concerning power-point-presentations are available at the website: www.sciencemath.ph-gmuend.de > teacher training.

The concerning material is summarised in the "module list" (see 4.1), also available on the website. The module list is a proposal consisting ScienceMath – modules to be offered in a Teacher Training event.

The Concept:
Structure:

- <u>Short, middle and variable versions</u>: Consists of E1 (see description below) according to the interests of the participants, perhaps in combination with other elements
- <u>Long term version: Consists of the five Elements E1, E2, E3, E4, E5.</u>
- <u>Long term version (with less university presence): Consists of the three Elements</u>
- <u>E1, E4, E5</u>

- **E1**: Presentations and workshops at the University – learning the content and material (ScienceMath-Modules, see detailed description below)
- **E2**: Intervening period – preparing the material for school
- **E3**: Workshop at the University – Discussing the preparation
- **E4**: Intervening period – implementation in the classroom
- **E5**: Seminar at the University – Exchange and improvement discussion

Description of the elements E1, E2, E3, E4, E5 in detail:

<u>Basis Element E 1</u>
The basis element is the starting activity for a long term version, the core of a teacher training event or the central part of it at the university resp. organising institution. It can be also offered as one single event (like e.g. in Ljubljana 2009).
Contents, length and intensity can be chosen according to the interests of the participants.

<u>Different possible versions of E1:</u>

<u>Introduction, Presentation/ workshop at the University</u>
<u>Version 1</u>:
Content: background presentation and one or more ScienceMath modules
Duration and kind of activities: afternoon, interactive presentation, discussion, group work, brain storming
<u>Version 2</u>:
Content: background presentation and one or more ScienceMath modules
Duration and kind of presentation: half or full day, interactive presentation, discussion, group work/ workshop with material, brain storming
<u>Version 3:</u>
Content: background presentation and different ScienceMath modules (individual chose)
Duration and kind of presentation: one or two days, interactive presentation, discussion, group work/ workshop with material, brain storming
<u>Version 4: European teacher event</u>
Content: background presentation and different ScienceMath modules (individual chose),
Duration and kind of presentation: one week, interactive presentation, discussion, group work/ workshop with material, brain storming

Examples:

Example for a two day event of version 3:
Day 1

▪ Welcome – registration and reception	½ h
▪ Presentation *ScienceMath* See Annex 1: prepared power-point	1 h
▪ Introduction into the theme groups See Annex 2: modules are according to the special offer	¼ h
▪ Break: Participants chose groups of special interest	¾ h
▪ First group meeting: Introduction into the theme according to the choice of the presenter	½ h
First activities, chosing cooperation partners	1 h
	4 h

Day 2

▪ Welcome - informal exchange	½ h
▪ Working in the groups Getting familiar with the content and material of the module/s	2 h

- Discussion in the group ½ h
- Break – lunch 1 ½ h
- Working in the group: Preparing own working sheets 3 h
- Working in the group: Arranging all needed equipment for teaching the module in school resp. writing a ToDo-list and planning the concrete school project; e.g. prepare a table with room for further remarks 1 h

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8 ½ h

Example (positive evaluated ScienceMath PD-event of Ljubljana 2009) of version 4:

Note: The following programme is a proposal and covering the already practiced event. The offer includes presentations of background ideas, modules, research results and workshops with material. The selection of the modules can change according to the target group and their interests. Material from the concrete held event is available on the website www.sciencemath.ph-gmuend.de > European Teacher Training Event.

Programme

Sunday

Arrival of the Participants for Teacher´s Professional Development

Monday

Time	Approx.	Session	Theme
8-12	15 min	Presentation	Welcome, Introduction
	45 min	Presentation	ScienceMath project presentation: Aims and Results
	30 min	Discussion	Participants introduce themselves
	15 min	Coffee	
	120 min	Presentation	Calculus; from physics …
12-14	Lunch		
14-17	180 min	Workshop	… towards mathematics
17-	Visit downtown		

Tuesday

Time	Approx.	Session	Theme
8-12	30 min	Presentation	Functions and Sounds
	60 min	Workshop	Functional relations Part 1: Introduction and trying the material - a
	15 min	Coffee	
	120 min		Functional relations Part 1: Introduction and trying the material - b
12-14	Lunch		
14-17	60 min	Workshop	Nutrition Circle, Proportions: Similarity and

			Allometry
	15 min	Coffee	
	90 min	Workshop	Creating interdisciplinary lessons between math and science; Part 1
17-	Free		

Wednesday

Time	Approx.	Session	Theme
8-12	60 min	Presentation	Experiments and concept of variable
	90 min	Workshop	Functional relations Part 2: Creating own worksheets
	15 min	Coffee	
	60 min	Workshop	Creating interdisciplinary lessons between math and science; Part 2
12-14	Lunch		
14-20 (or 13 - 20)	360 min	Excursion (Workshop)	Measurements in real world – field work incorporated into science teaching excursion

Thursday

Time	Approx.	Session	Theme
8-12	30 min	Presentation	Concept of parallelism and concept of gravity
	45 min	Presentation	Fermat meets Pythagoras and Fermat´s Principle
	30 min	Presentation	Parabola and Technology
	15 min	Coffee	
	30 min	Presentation	Students’ discussions about mathematics and society: Modelling population growth.
	60 min	Workshop	Functional relations Part 3: Discussing the module
12-14	Lunch		
14-17	90 min	Workshop	Arithmetic mean and car differential
	15 min	Coffee	
	60 min	Presentation	Logarithms
17-	Typical dinner		

Friday

Time	Approx.	Session	Theme
8-12	30 min	Presentation	Modelling motion: the case of shooting in water
	60 min	Presentation	From coupled pendulum toward Fourier analyses

	15 min	Coffee	
	90 min	Discussion	Final discussion
	30 min	Conclusion and farewell	

E 2
Intervening Period: Preparing the materials for school

- The chosen modules are prepared for school: Preparing/ buying the needed equipment, copy of worksheets etc.
- Thinking about a concept of implementation: room, class, cooperation partner etc.
- Documentation of experiences and critics

About: 1 month time

E3
Workshop at the University

Day 1

- Welcome – registration ½ h
- Working in the theme groups:
  Exchanging and discussion experiences and material,
  If needed: preparing and improving the material for implementation 2 ½ h
- Common lunch 1 ½ h

---

4 ½ h

E 4
Intervening Period: Preparing the modules and Teching them at school

- The chosen modules are prepared for school: Preparing/ buying the needed equipment, copy of worksheets etc.
- Thinking about a concept of implementation: room, class, cooperation partner etc.
- Doing the necessary steps for realisation
- Teaching and implementation at school
- Documentation of experiences and critics

About: 8 weeks

E 5
Seminar at the University

Day 1

- Welcome – registration and reception ½ h
- Working in the theme groups:
  Presentation of the results/Experiences and discussion 2 h

- Break: lunch 1 h
- Working in the group: Discussion , resp. improving the material 1 ½ h
- Break: coffee and cake ½ h
- Exchanging the material between all participants 1 h
- Plenary: Short Presentation of the modules ideas, concerning important aspects for a successful teaching 1 ½ h
- Informal close – time for more exchange and further plannings 1 h (open)

9 h with open end

Continuing the same way with other modules.
Individual changes are of course possible.

2.2.3 Framework "Math and Science under one Roof"

Core of the ScienceMath-Professional Development concept are of course the ScienceMath modules. The included ideas stimulate cross-curricular lessons, the prepared worksheets help directly to realise interdisciplinarity in the classroom. The intervening periods of the PD-concept e.g. offer the possibility for concrete planning and implementing and, if needed, for assimilation to the specific needs of the class.

But before planning there is the task of chosing a module. The "roof" (Zell 2010, figure 9) offers a quick overview over a teaching module, especially over the share of science and math, the potential for mathematical literacy and organisation aspects. Additionally it is an analysing instrument for ready courses and allows their evaluation (e.g. in respect to the potential of heuristic competencies etc.).

It follows a description of the "roof"; in 3.2.2 it is described at an example. In Annex 2 you´ll find roofs for the ScienceMath modules.

Description of the "roof" (compare figure 9):

At the very top a short overview of the lesson and the suitable age is given. Then the theme to be taught is seen in three different views: content, heuristic competencies and organisation.
Looking at the upper parts of the three columns, notes give a quick overview of the mathematical and scientific contents involved, the pre-knowledge required (content), heuristic competencies touched, way of teaching (heuristic competencies), materials needed and preparations to be done (organization). The lower parts of the three columns give a deeper view of these elements mentioned in the upper part. In the column *contents* interdisciplinary views of the mathematical and scientific concepts are given. For each concept mathematical, common and outer-mathematical aspect of a concept can be described. In the lower part of the column *heuristic competences* the heuristic competencies involved are exemplified, common aspects resp. subject-specific aspects can be mentioned there, too. The lower part of the column *organization* shows the portion of math and other teachers involved to communicate and coordinate. The portions give an estimation how much preparation should be done in advance of this lesson. Of course this estimation is quite rough and depends on the teachers involved. At the very bottom there

is space for further comments, which are seen important for this lesson and don't match with one of the three columns.

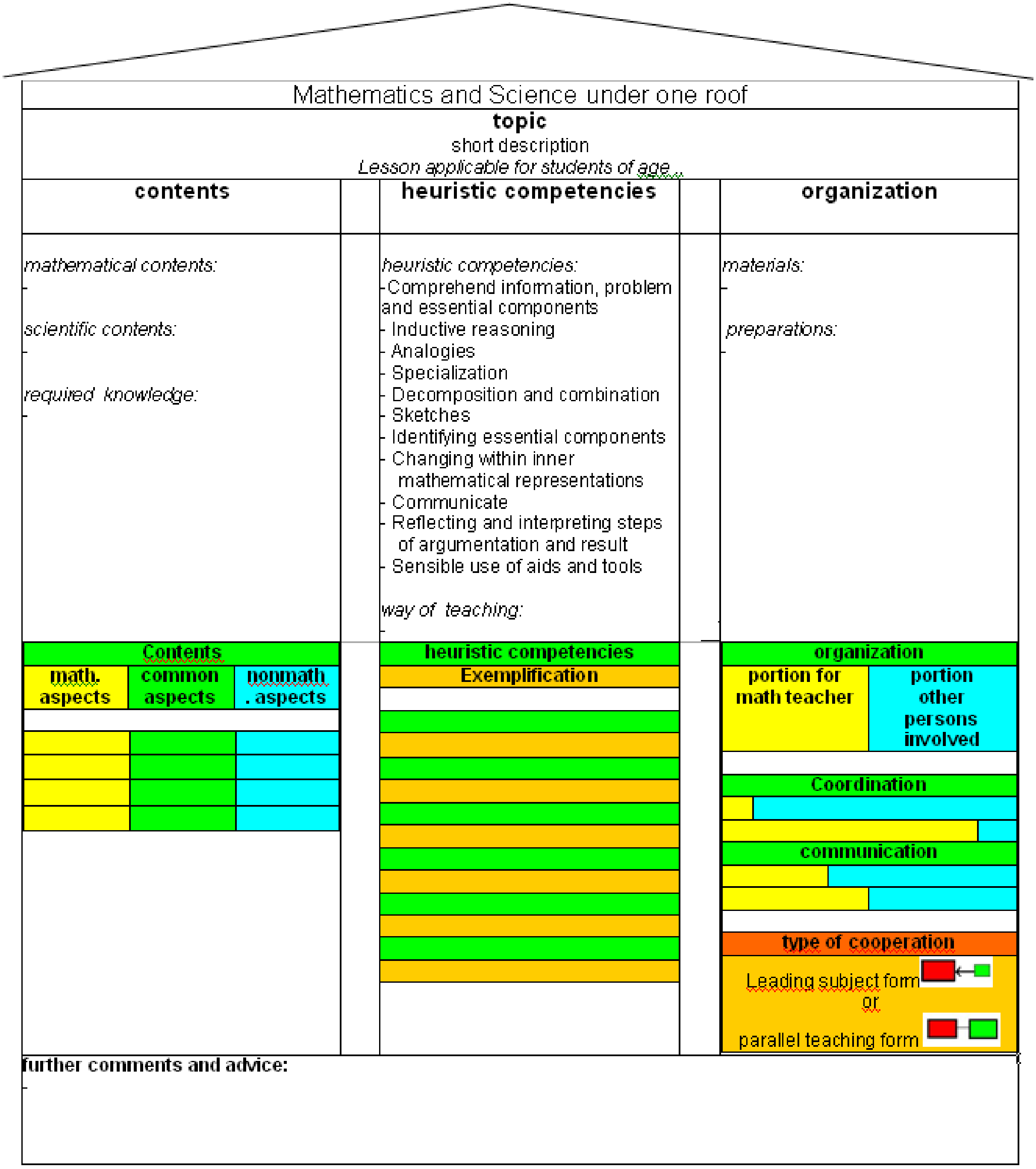

Figure 9
The "roof": Math and Science under one roof (according to Zell 2010)

2.2.4 Conceptional Concidecations for Benefiting from Interdisciplinary Activities

The objective of cross-curricular teaching must be *enrichment*. It should lead to a support in learning contents, methods, special competences or ways of thinking. The decision to teach in a cross-curricular way must therefore be preceded by an analysis of its objective, which ensures (to a large extent) that enrichment is part of it. In detail, this requires e.g. the consideration that acquiring a subject competence is made easier through alien-aspects (2.1.1). This requires, however, a basis for such judgement, such a higher ranking criteria, that are applicable to the respective individual case. At the same time, the higher ranking criteria form a rationale for cross-curricular teaching and arise, apart from subject aspects, from different disciplines such as education, psychology and the theory of science.

We will now present the objectives of cross-curricular teaching. You´ll find an explanation under
www.math.umt.edu/TMME/vol6supp/TMMEvol6_supplement1_March2009.pdf
where the whole conceptional framework for cross-curricular teaching is published (Beckmann 2009):

Cross-curricular/ interdisciplinary teaching

- as an opportunity for students´ orientation,
- as a field for holistic learning,
- as a particular opportunity for motivation,
- as field for a new way of thinking,
- as an opportunity to reflect on subject-specific methods,
- as a "counterpart" to specialisation,
- as an (additional) opportunity for learning important basic mental techniques,
- as a field in which to experience the social reality of science,
- as an aid in integrating and structuring learning,
- as a field for the improved practice of general competencies,
- as an opportunity to develop ways to deal with heterogeneity,
- as a contribution to general education,
- as a special opportunity to deal with topical issues,
- as a special opportunity to disclose the importance of interdisciplinary cooperation in the solution of problems,
- as an opportunity to solidify subject knowledge,
- as an opportunity to experience the particular importance of a given subject,
- as a special opportunity to tackle the problems of a particular subject.

To explain e.g. the last point and forge link to the ScienceMath project we relate to TIMMS/PISA:

> "The deficits in the achievement of German high school student compared with students from countries with higher student achievement are particularly obvious in tasks that test mathematical-natural science understanding, demand a flexible application of learned material or offer an unusual problem constellation." (Babtist 1998, p.5)

The weakness show up

- if a flexible connection of several subject areas is required,
- if several steps have to be combined to solve a task,
- if different aspects of a topic are addressed simultaneously,
- if the use of unaccustomed material is required, i.e. terms not used in the accustomed contexts,
- if the construction of complex models is expected. (Neubrand et al. in Wiegand 2000, p. 95).

Organisational and methodological consequences have been drawn from these results. The ScienceMath modules should help to realise a better and more successful teaching culture.

3 Development and Research Results

3.1 Conception of the Modules

3.1.1 Involved Subjects

Mathematics has connection to all subjects and fields of reality – or to say it in the sence of Freudenthal: Mathematics is the core subject which attract objects of other subjects so that students should plough it like fields which have to be arranged mathematically (Freudenthal 1978).

Of course there is a special connection to science. Sciences and mathematics include similar methods like deductive and inductive thinking. Furthermore the application of mathematics is natural in the sciences, especially in physics. Scientific contents lead to modelling activities in a way mathematics could be experienced meaningful and through typical mathematical methods.

The ScienceMath teaching modules are prepared for interdisciplinary mathematical lessons resp. interdisciplinary lessons between science and math. Most of the modules connect mathematics and physics; but there are also many modules supporting cooperation with biology (e.g. *nutrition circle, paramecia)*, but also with chemistry (e.g. *relation between math and volume)*, technics (e.g. *arithmetic mean and car differential, parbola and technology*) and even geography (e.g. *logarithmic function, growth*). Most modules start with an impulse from reality or an experiment carried out in "reality" like real time measurements or measurements at the crossroad. This includes the chance of authentic experiences, linked learning and a meaningful and adequate understanding of mathematics and its concepts (compare also 1.2).

3.1.2 Teaching Methods

The ScienceMath modules offered in this volume resp. on the project´s website www.sciencemath.ph-gmuend.de should lead to a new teaching culture in Europe. The close connection to reality resp. to science contents and the mean of different kind of experiments should support own and authentic experiences and intuitive learning. If there are teacher-oriented phases necessary, they should take place in an open atmosphere with a constant exchange between teacher and students. Important is the opportunity for self-dependend work which is motivated by worksheets. In many modules stations or the change in perspective in different parts of the module should lead to linked learning and to comprehensive experiences.

3.1.3 About Supporting Mathematical Literacy

It is a stressed aim of the ScienceMath project to support mathematical literacy. In Annex 2 the ScienceMath modules are analysed and presented through the "roof" (2.2.3), which allows a quick overview over the support of the module to heuristic thinking. According to the description in 1.3 heuristic thinking is an important element of mathematical literacy.

A specific element in practicing the ScienceMath modules is the use of experiments. Experiments have to be seen as an important aspect in supporting mathematical literacy. Here our argumentation follows the conception of mathematical literacy as described in 1.3.1 (according to Zell, Zell 2009). This conception allows investigating the support of mathematical literacy of teaching material or methods in an appropriate and convenient way. According to that conception one has to consider heuristic competencies which support heuristic thinking and deductive reasoning. Mathematical concepts and procedures should be learned functionally, intergratively and multifacetedly. Experiments can touch many aspects of mathematical literacy as following there sections will show (Zell 2009,a):

Heuristic thinking: Heuristic thinking is helpful to conceive problems/contents, to interpret or solve if necessary. Investigating phenomena by experiments affords recognizing and structuring them. That's why heuristic thinking processes have a leading role. Let's go into detail:

- *Comprehend information, problem and essential components*
  Since quantitative experiments contain mathematical concepts they are also math problems. Therefore heuristic strategies are applied. They aren't limited to mathematical aspects like evaluating data, but can also applied to identify scientific concepts and processes. In particular:
  - *inductive reasoning*
    Inferences done with measured data of quantitative experiments are done inductively. This process is the same as in mathematics when patterns have to be found.
  - *analogies*
    To conceive and structure a phenomenon one often tries to explain it with concepts and methods already known, like in mathematics. The process is the same, but the theoretical background differs. That's why commonalities are not that apparent.
  - *specialisation*
    One characteristic method conceptualizing an experiment is controlling variables. This is equivalent to the strategy of specialisation.
  - *decomposition and combination*
    If problems contain more than one factor, decomposition in different experiments is a good way to find relationships among the factors involved. After that these relationships are combined to a whole; e.g. resistivity of a conductor.
  - *sketches*
    Like in mathematics sketches can be very helpful in experiments because they allow reducing a phenomenon or an experiment to its main components. Sketches may differ because of looking at the phenomenon through subject-driven eyes. The essential, that means visualizing a problem by its characteristic components as a basis for further investigation, is the same for both subjects.
  - *identifying essential components*
    Both science and mathematics try to explain information with as few as possible components to gain rather simple models or relationships. Especially physics and chemistry ignore concepts that are involved but whose effects are negligible, e.g. buoyancy in air.
- *Changing within inner mathematical representations*
  To gain and evaluate data different mathematical representations are applied. This is to better visualize data and finding relationships, since those may be more obvious in another representation.
- *communicate*

The ability to communicate in a meaningful way is essential in any field. But communicating during quantitative experiments also means (implicitly) communicating about mathematical contents.

- *Reflecting and interpreting steps of argumentation and result*
  To get reliable data and relationships, reflection, how and why instruments are used and if steps of argumentation are reasonable, is necessary. Interpretation of an experiment is necessary to integrate a phenomenon in given theory and can be a basis for new hypotheses.
- *sensible use of aids and tools*
  Aids and tools in science involve more than those in mathematics, which are to evaluate data. But knowledge what aids and tools are available, how they work and what range of application and limits they have, is necessary too; even more important than in mathematics.

By applying scientific experiments many heuristic competencies, which support heuristic thinking, can be touched. But one has to consider that those competencies have different peculiarities, because of their theoretical backgrounds. The essentials however are the same. Phenomena have to be described and structured by given theory, this means identifying essential components and possible relationships among them.

Comprehensive understanding of mathematical concepts and processes: In most scientific experiments mathematics is involved. Therefore it may contribute to mathematical literacy. To be a contribution to mathematical literacy mathematical concepts and processes should be taught in an integrative, functional and multifaceted way. Obviously teaching mathematical concepts by experiment is functional. It is also multifaceted since expanding the domain offers new views to be experienced. Sometimes different aspects of a mathematical concept can be touched. How multifaceted the aspects of the concept of variable can be touched by physical experiments is shown in (Zell 2008). By experiments students can connect mathematical concepts to concrete objects already known. This makes it easier to store knowledge and better integrate it in given theory. Sometimes the same concept can be involved in different experiments. Then students can make new connections to already known concepts and can reflect their perceptions regarding these concepts.

Familiarity with deductive reasoning: Especially physics and chemistry are deductive theories. Inferences don't have to go into detail like in mathematics but hypotheses deduced from theory have to be in accordance to experimental results. This means if experiments are done to confirm a hypothesis, these hypotheses were inferred deductively. If generalisations are made after the experiment, these will also involve deductive reasoning. Therefore familiarity with deductive reasoning is a necessary component if doing experiments. Considering a mathematical point of view experiments have a great potential to improve deductive reasoning since inferences can be mostly done on a descriptive level.

3.2 Experiences and Research Results

3.2.1 Introduction

An important objective of the ScienceMath project is/was not only the development of teaching modules, but also the investigation of the realisation in the classroom and the learning support. In the project different research activities were carried out: Teaching and observation in the classroom, documentation of experiences, systematic video and audio reports/ transcriptions, evaluation of working sheets, questionnaires and interviews.

The results are presented at teacher trainings/professional developments and conferences. They are published in scientific and teacher´s journals, in proceedings etc. (for references see 5 and the website www.sciencemath.ph-gmuend.de > literature). In this chapter we will summarise some of the results exemplary.

3.2.2 The Modules´ theoretical Background

> Questions of interest: Which concept is learned in the developed sequence? Which concepts´ aspects are touched in the sequence? In which way do the students act with these aspects? Is there the possibility for an intuitive understanding, for authentic experiences?

In this chapter we summarise results concerning the modules´ theoretical background. A central mathematical concept which is focussed in many of the ScienceMath- modules is the concept of function. Functional thinking implies a multiplicity of capacities. Thus, various representation forms of functions have to be mastered (e.g. graphs, verbal descriptions, algebraic expressions, tables) and transformed from one into another (Swan 1982). In addition, the functions must be viewed in such a way that they are suited to the solution of the problem. When applicable, students should be able to look at them as the classification of points (so called action layer), a dynamic process (process layer) or even as an individual object that can be manipulated (object layer) (Sfard 1991; DeMarois & Tall 1996; et al). Our ScienceMath-partner Höfer illustrated these capacities in a *House of Functional thinking* (Höfer 2008, figure 10). The *House of Functional Thinking* is a huge tool for analysing students´ competencies, school books, curricula and teaching modules as far as the concept of function is concerned (Höfer 2008). This will be shown at examples in 3.2.3.

Functional thinking has to be seen as a basic competence for a successful (mathematical) learning. That´s why many ScienceMath-modules concern to this concept resp. to the concept of variable. Examples are the modules *Proportional factor 1 and 2, Functional relations 1, 2 and 3, temperature, relationship between mass and volume of a liquid, decaying processes, Boyle-Mariotte, bouancy, refraction, thermal expansion, GPS, growth* and more. Specific functions are investigated in the modules *sound functions, parabola, logarithmic functions, paramecia, function x^(3/2), x^(-0,5), growth* and more. As well there are modules with concern to functions on a higher level or with a stress on modelling processes. Further mathematical concepts touched in the modules are e.g. the concept of parallelism, similarity, arithmetic mean, calculus, arc length etc. A summarising overview you´ll find in the module list in 4.1, including all English translated modules of the project.

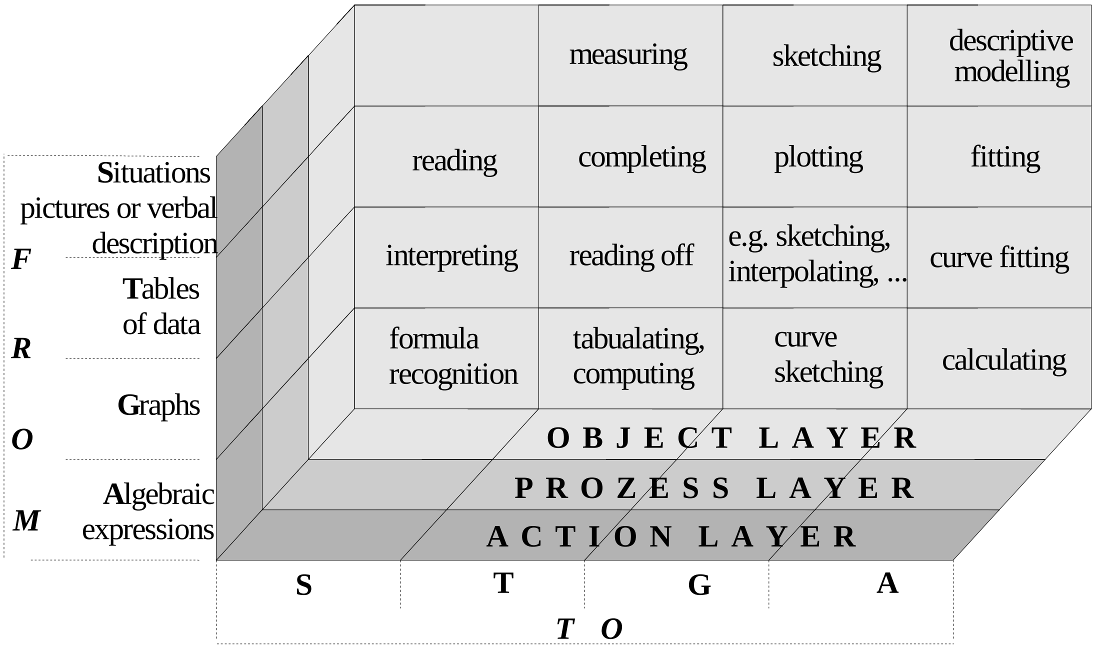

Figure 10
The House of Functional Thinking (Höfer 2008)

A description from another perspective offers the “roof” (see 2.2.3) as a result of a detailed analysis concerning mathematical and extra-mathematical contents (concepts, methods etc.), touched heuristic competencies and interdisciplinary organisation (worked out by Simon Zell, Zell 2010). The “roof” allows a very quick overview over the module (figure 11).

Applying the “roof” to the module *Thermal expansion of a liquid:*

> After a short description of the lessons, an overview about the contents involved, the heuristic competencies touched and the materials needed is given. After that the interdisciplinary potential is shown. Both subjects use variables as representatives of quantities. From a mathematical view variables are quantities which are first of all context-free, whereas in physics variables are embedded in physical theory. Therefore the names of variables are based on conventions. It is the same if you look at the functional relationship, mathematical interest in rather on the structure whereas physics looks at the relationship contextually. Now looking at the proportional factor: in physics it is a coefficient of thermal expansion; in mathematics it is seen as a specific unknown. Both subjects have to consider measuring errors, but their concerns differ. Math is rather concerned about the validity of the results. Physics is rather concerned about the origin of measuring errors. Looking at the heuristic competencies you can see that doing experiments touches competencies common to both subjects and allow critical thinking about the phenomenon and the results. In the third column it is shown what should be done for coordination and communication. The portion for math and physics teacher is roughly estimated. Underneath it is shown that this lesson can be taught in Leading subject or parallel teaching form. At the very bottom, further comments are given, which result from experiences in class or are seen important to this lesson.

In chapter 4 you´ll find the specific roofs to the modules and many of the Science-Math-modules in detail (including background information, materials like worksheets for the direct use, and further information e.g. experience reports etc.). All developed modules are available for direct download at the website www.sciencemath.ph-gmuend.de (click a flag in the specific language). Some of them are motivated by videos as well.

Mathematics and Science under one roof

Thermal expansion of a liquid and concept of variable

Students measure heights of a thermometer without a scale at different temperatures. They find a relationship between the difference of height and difference of temperature (proportional) and discover the properties of that formula, especially different aspects of the concept of variable

Lesson applicable for students of age 12-14

contents	heuristic competencies	organization
mathematical contents: - variable as generalised number - variable as specific unknown - variable in a functional relationship *scientific contents:* - thermal expansion of liquids *required knowledge:* - handling measuring errors	*heuristic competencies:* - induction - recognizing essential components - changing within innermath. Representations - communicate - reflecting/interpreting *way of teaching:* - working in groups	*materials:* - thermometer without scale - electronic temperature measuring device - beakers - kettles *preparations:* - if possible doing experiment in advance

Contents

math. aspects	common aspects	nonmath. aspects
general contextfree quantity	variable	certain letters indicate certain physical quantities
quotient between 2 variables constant	fct. relationship T↑, Δh↑	contextual
specific unknown	proportional factor	coefficient for expansion
relationship still valid?	measuring errors: quotients never equal	origin of measuring errors?

heuristic competencies

Exemplification

induction
finding proportional relationship out of several pairs of measuring values
recognising essential components
which characteristic elements determine relationship?
changing representation
from measuring values in table, describing fct. Relationship verbally then algebraically by a formula
communicate
which relationships appear; validity of formula, discussion about measuring values
reflecting/interpreting
validity of formula; range of variables

organization

portion for math teacher	portion other persons involved
Coordination	
access to material in physics room	
enough plugs in class room?	
communication	
	students' problems doing experiments, maybe explaining physical background
	Do students have experience in handling measuring errors?

type of cooperation

Leading subject form

or

parallel teaching form

further comments and advice:

- at task 5 help students finding formula, if necessary
- combination with buoyancy and Boyle's law experiment possible, if not enough material for whole class
- if students have no experience in handling measuring errors, give short introduction into that topic in advance and remind students that measuring errors appear when they try to find the formula.
- main emphasis on mathematical aspects; discussing physical aspects after that lesson or in physics class if one wants to do so
- teaching material prepared as leading subject form, can be modified easily to parallel teaching form

Figure 11
The “roof”; example: Module *Themal expansion of a liquid*, see www.sciencemath.ph-gmuend.de > teaching material

3.1.2 Learning Activities of the Students

Questions of interest: How do the students connect reality/science and mathematics? Are there modelling activities initiated? Which? In how far do the students experience mathematics as something important?

Some of the ScienceMath-modules are developed upon the specific aim to stress modelling activities. As described in 1.4.3 (example *Modelling things in traffic)* the modules motivate a modelling process which connects reality, real model, scientific model and mathematical model. Modules like *growth* etc. stimulate a change in perspective which corresponds to a change in status from a situation (e.g. radioactive decay or reproduction of bacteria) to a model of a mathematical concept (e.g. exponential function).

Of course all modules between science and math include modelling activities: A real situation has to be described with mathematical means to explain it or to allow prescriptions. Systematic observations in the classroom or in the laboratory show interesting results. E.g.:

The following result concerns the module group “Functional relations” and here the experiment “tunnel”. The students worked out the relation between the intensity of light and the distance from the light source by using a lux metre to measure the light in tubes of different lengths (figure 12).

Figure 12
Relation between intensity of light and distance from the source of light

In the following quotation of their final report, the students in this group used the reality (tunnel), the realistic model (tube) and the mathematical model (graph) in parallel. They used the graph to explain the light conditions in the tunnel. They described the relation of 9.7 cm to 37 lux and 30 cm to 0.1 lux. In doing so, they also formulated the reciprocal relation (long to dark, short to bright). Thus, they thought in parallel in nearly all the models without any noticeable signs of difficulties in the changes of representation. An equally unproblematic switch between the action layer (in that single pairs of values are considered in the argumentation) and the process layer (the further away from the mouth of the tunnel the darker it becomes) can be noticed:

> *“We called the second project ‘light in a tunnel’. The further the car drove into the tunnel the darker it got. The first tube had a length of 9.7 cm. When we held it to the window, the intensity of the light was approx. 36 lux. When we held a 30 cm tube to the window, we only measured 0.1 lux.*

In an extra graph[4] we were able to establish exactly how much light there was at the entrance to the tunnel and how much there was left at the end of the tunnel."

A typical mathematisation method is the use of variables. A central part of the ScienceMath project relates to the systematic investigation of the use of variables while doing experiments. The following results concern to the experiments *Buoyancy* (proportional relationship between the forces in air and water), *Thermal expansion of a liquid* (proportional relationship between difference of height of an uncalibrated thermometer and difference of temperature) and *Boyle-Mariotte* (inverse proportionality between air pressure and position respectively volume). After measuring at least six different values, students are asked to describe the relationship first in their own words and then through a formula. This formula shall then be used to calculate measuring values. These values shall be checked by looking at the before measured values. Observations show that the students were indeed doing modelling activities. They described the experimental situation mathematically (Zell et al. 2009):

Students can identify the measurands with their chosen variables. A few examples:

Buoyancy experiment:

I2: Can you tell how you recognize (the experiment in your formula)?

S6: yes, you see the statement for air and for water. And yes the result, yes…

Here the group chose word variables. If they didn't choose words they chose the units of the measurands.

Boyle-Mariotte experiment

I1: what are those cm? What do they stand for?
S1: mmh here at that strip for example 6cm
I1: mmhmm
S1: so for the respective number
I1: and the x?
S1: for the respective pressure

Here the student chose the units of the physical terms as the name of his variables. Since he didn't know the unit of pressure, he chose x.

Some interview episodes show that the students have done the first part of the modelling cycle by examining the phenomenon and structuring in a formula. That has been done in different levels. Weaker students could only explain in words and the strongest even presented three equivalent formulas.

Buoyancy experiment:

S4:Then we agreed, that if you divide air by water, the result is always the same. It doesn't matter, if there are 1,2,3,4,5 cylinders. The result is always 1,2.

Boyle-Mariotte experiment:

I1: What have you found out?

[4] A graph matching the test results is intended here.

S1: yes, that device. If you turn further that thing moves forward and the further it moves the measuring number get smaller and the pressure get higher.

Thermal expansion experiment:
S17: We had to find formulas. These were height times x is difference of temperature and difference of temperature divided by x is then height and difference of temperature divided by height is then x. [...]
I8: and what changes in general in your formula?
S17: temperature and the head of liquid there, both get higher the more water you add.

3.1.3 Concept Understanding of the Students

Questions of interest: Which previous experiences in mathematics and science do the students have? What are their attitudes towards these school subjects? How do the students explain the basic concepts using their own words – before and after the sequence? Is there a conceptual change? Does the sequence support an intuitive understanding and authentic experiences according to mathematical (scientific) literacy?

We will show exemplary some results very shortly. For more information please read the referred papers.

The following summary concerns the concept of function, published in (Höfer 2008) and in (Michelsen et al 2007)[5]. Some results to the concept of variable are already described in 3.1.2 (and see Zell 2008). We will start with an initial consideration (Höfer & Beckmann 2009): While New Maths was taught in schools way into the 1980s, functions were developed from set theory. For the students they were static constructions whose dynamic character was not fully understood (Sfard 1991). In addition the algebraic treatment and analysis of functions was the main focus of the tuition that was based on this static conception of functions. This one-sided perception of them as a problem of teaching functions in the classroom was still described in papers published way after New Maths (Cunningham 2005, PZ 1990). If one keeps to the graph of the House of Functional Thinking one notes that traditional teaching has put an emphasis on the action layer in the area of the right column (translation into algebraic expressions) and in the bottom line (translation from algebraic expression). This one-sidedness repeatedly causes deficits in the problem solving competence in the field of applied mathematics because the non-algebraic capacities – in particular the interpretative and graphic ones – are not well enough developed (PZ 1990, Sfard 1991, Cunningham 2005, et. al.)

Classroom investigations about ScienceMath modules show positive tendencies.

E.g. in the run of the experiments (radioactive decay, bacteria growth etc.) and the construction of adequate models the students have to understand functions as processes as well as objects. From a student´s experience report (Michelsen et al. 2007, p. 55):

"$N_a = 1000 \cdot (1-1/6)^t, N_b = (1000 - 1000 \cdot (1-1/6)^t) \cdot (1-1/3)^t$ und $N_c = 1000 - N_a - N_b$. The first factor of N_b represents the number of A-nuclei, that decay to the nuclide B. The second factor represents the number of B-nuclei, that do not decay to C. The sum of the nuclei over the

[5] Some of the texts are direct quotes from these papers.

whole time is 1000; that´s why the number of B-nuclei is 1000 minus number of A- and B-nuclei."

The next quotation from a students´ report indicates as well, that context related experiments support the object aspect (object layer/ precept). Formal and concrete model are linked:

"We saw that cooling of a hot liquid can be described by an exponential falling function. That´s why the graph has the form $y = ba^x + c$. In the formula c should be the room temperature; instead we should have to cool the liquid with something else. The graph describing the heating of the ice water was quite opposite to the graph of cooling. If you place the temperatures of both graphs at the same time line, you´ll get a horizontal graph."

Höfer tested the effect of experiments to functional relations (see module group *Functional relations*) on the concept understanding. The developed questionnaires include mathematical tasks relating to the different transfers in the house of functional thinking (3.2.2, Höfer 2008). Four classes of 8th graders were tested before and after teaching the module with similar taskgroups concerning the transfers in the house of functional thinking. The test results were evaluated by statistical methods. For reasons of space we describe the results without any detail and only by illustration in the house of functional thinking (figure 13). For more information we refer to the original literature, especially to the detailed research report of Höfer (Höfer 2008).

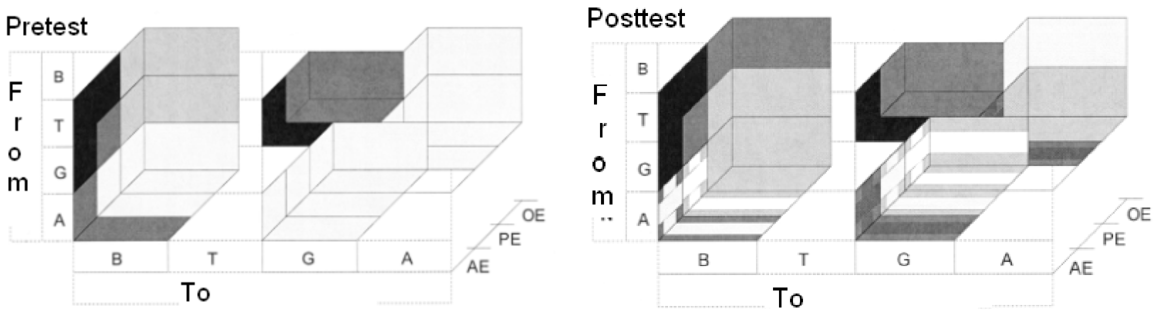

Figure 13
(Höfer 2008, p.121),
Results of the pretest: the darker the bricks the higher the ratio of solutions,
Results of the posttest: striped blocks show significant improvement (according to the legend in figure 14)

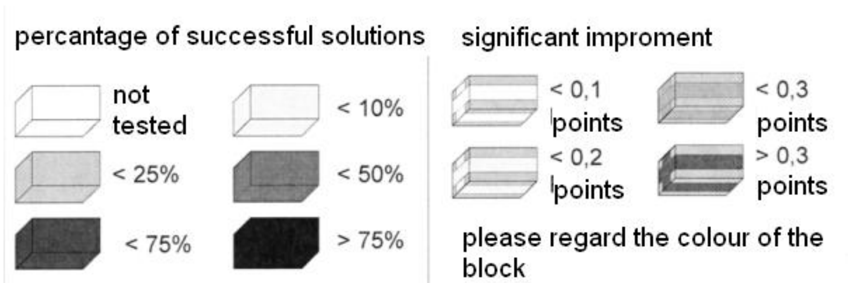

Figure 14
Legend to figure 13

Figure 13 shows the results of pre- and posttest. Before experimenting to functional relations the students already solved some of the tasks on a very high level. For those tasks it was – of course - difficult to measure improvements. Significant improvements could be observed when the solution was on a lower level. It is remarkable that this appears especially in connection to algebraic descriptions of functions. Even on the object layer the results show improvements at transfers from algebraic term to graph and vice versa.

The question of conceptional change is a research line at our Finnish partner university. In her previous papers Kaarina Merenluoto stresses that conceptional change is more difficult when it needs a drastic revision or restructuring of old knowledge (Hannula et al., p 195, Merenluoto & Lehtinen 2002, Merenluoto & Pehkonen 2002). Experimenting with its modelling activities offers an easier way for conceptional change. This is because of the systematic and comprehensive occupation during the scientific procedure, which includes the proving of hypothesis through drawing and interpreting graphs etc. – as shown in the following examples:

> *Relation between intensity of light and distance from the light source*
> S1: Normally it is not proportional
> S2: Ye, then let´s do it; then we know
> S1 draws their measured two points into a coordinate system
> S2: That is not proportional
> S1: Sure?
> They understand that they have to measure more and start a systematic measurement before proving their hypothesis.
>
> *Relation between air pressure and air volume*
> S1: Then we´ve drawn the measurements into the coordinate system. Most of us think that it is inverse proportional.
> S2: Because of the way of the line
> S3: When air pressure grows, the tubes decrease. (Beckmann & Litz 2009)

3.1.4 Realisation in the Classroom and Evaluation through Teachers

> Questions of interest: Is it practicable to teach the sequence? What are the positive characteristics of it? Does the sequence support learning of the aspects of the concerned concepts? Does the sequence support other aspects concerning the learning of mathematics? Which? Which aspects support cooperation between math and science teachers? What do teachers think about the sequence; would they teach it? Is it practicable to teach the sequence? What are the positive characteristics of it?

Key-person in this research line is the teacher. During the project run the reactions of teachers were and are important for our development work. In different teacher trainings or in individual interviews resp. written questionnaires we tried to get an overview over the chance of implementation of single modules in the classroom and the practicability of the modules from the sight of the teachers. The interviewed teachers learned about the modules in paper form, in a presentation or directly by trying the material. Apart there was a special investigation about interdisciplinary teaching in general (see 2.1.3). Summarising, ScienceMath modules got a very positive feedback. This includes questionnaires especially at the European teacher training event in Slovenia 2009. Additionally we expect more return information after the teachers realised the modules in the classroom. There are promises for that.

Some comments:

"Great idea; I like the simple experiments; the students have to act"
(teacher´s comment in a questionnaire after a one day teacher training event)

"concern to applications", "motivation", "ingenious", clarity and applicability", "Context is not only explained; it is captured", "motivating mathematics in general", "many examples from every day life", "linking math and physics"
(teacher´s key-words to the question which aspects are supported through the ScienceMath module)

"Remarkable is the long-term effect. Later in the school-year the students looked back to the functional relation module and argued referring to it." (teacher who used the experiments in his 8th grade glass).

Apart from the teachers we used questionnaires for students and talked with them about the modules and their experiences.

Examples:
"I liked it, because we could do experiments and were allowed to try something out."
"The best, in my opinion, was the experiment buoyancy; that there is less force in water than in air and there was the lowest force in salt water."

"At Experiment 9 we thought it was physics, but it was mathematics. We saw physics and mathematics have many aspects in common."

"Sometimes you should use experiments for learning."

"Normally interdisciplinary mathematical lessons are not really interdisciplinary. But in this example the subjects play together very good and the connection was obviously."

For implementation in the classroom and for evaluation activities teacher trainings resp. professional developments PD are very important. The European teacher training event in Ljubljana 2009 was a test run, which was positively evaluated. 100 % of the teachers "strongly agreed" that the ideas presented at lectures and workshops were interesting, clearly and well presented, knowledgable and well prepared. Nearly all of them felt able to use them in their future work.

4 The Modules

4.1 Overview

In the three years of the project run more than 30 modules could have been developed in detail. Our first sequences initiated a creative development process which is still ongoing. The idea of the ScienceMath approach is still in the mind of the partners, so although the EU support is finished, the development activities are continuing.

We tried to realise, that all modules are available in English as this language is used international. We experienced also that English is a very known and used language in the Scandinavian countries. Most/all modules are translated into the languages of the ScienceMath-partner-countries (Slovenian, German, Finnish and also some Danish) as well as into Spanish, French and Turkish. You will find it on the Website www.sciencemath.ph-gmuend.de.

ScienceMath - Module list (all English translated modules)

Lower to middle secondary level

Topic/ Theme	Age and Key-words	Special (interdisciplinary) approaches	Subjects involved and cooperation form
Concept of Parallelism	10 -12 years, parallel, distance, concept of equal distance	Simple experiments	Mathematics (resp. physics), TM
Temperature	10 – 15 years, Variable, change, rate of change,	Simple experiments	Mathematics, chemistry, TM
Small Car - Accleration	10 – 16 years, Velocity, speed, safety in traffic situations	Simple experiments with toys	Mathematics, Physics, TM or JP
Experiments to Proportional Factor 1	12 – 15 years, Proportionality, proportional factor, concept of function, functional relation, linear function	Simple experiments introduced by realistic situations	Mathematics, resp. physics, TM
Physical Experiments to Proportional Factor 2	12 – 15 years, Proportionality, proportional factor, concept of function, functional relation, linear function, Physical experiments in interdisciplinary lessons	Physical experiments introduced by realistic situations	Mathematics, physics, TM or JP
Boyle's Law and Concept of Variable	12 – 16 years, Concept of variable, modelling, functional relationship, inverse proportional relationship, Boyle's Law	Physical experiment introduced by realistic situation	Mathematics, physics TM
Buouyancy and Concept of Variable	12 – 16 years, Concept of variable, modelling, functional relationship, proportional relationship, buouyancy	Physical experiment introduced by realistic situation	Mathematics, physics, TM
Refraction and Concept of Variable	12 – 16 years, Concept of variable, modelling, functional relationship, proportional relationship, refraction	Physical experiment introduced by realistic situation	Mathematics, physics, TM
Thermal Expansion of a Liquid and Concept of Variable	12 – 16 years, Concept of variable, modelling, functional relationship, thermal expansion of a liquid,	Physical experiment introduced by realistic situation	Mathematics, physics, resp. chemistry, TM or JP
Functional Relation 1	12 – 17 years, Function, linear, quadratic, cubic, inverse proportional and other functional relations, contexts in reality – simple experiments in mathematical lessons	Experiments with long worksheets introduced by realistic situation	Mathematics, resp. physics, TM
Arithmetic Mean and Car Differential	13 years and older, Arithmetic mean, car differential	Technological,	Mathematics, technics, JP
Fermat meets Pythagoras	13 – 15 years, Pythagoras' Theorem, extreme value tasks, Fermat's Principle	Physical experiments	Mathematics, physics, PP

Relationship between Mass and Volume of a Liquid	13 – 15 years and other, Variable, proportionality, constant, function, unit, density	Experiments, modelling	Mathematics, chemistry, TM or JP
Nutrition Circle and Pie Charts	14 -16 years, Pie Charts, Percentage calculation, Nutrition Circle, Nutrition	Modelling	Mathematics, biology, TM, PP
Proportions: Similarity and Allometry	15 – 16 years, Similarity, homothety, allometry, relation surface and volume, Appearance and behaviour of animals	Integrated worksheets	Mathematics, biology, JP

Middle and higher secondary level

Topic/ Theme	Age and key-words	Special (interdisciplinary) approaches	
Similar Triangles for Paralax Measurements	14 – 17 years, Similar triangles, parallax measurements	experiments	Mathematics, physics, TM or PP
Center of mass/ gravity	14 – 19 years, Intersection in triangles, center of mass in planes and bodies, applications of linear algebra, gravity	Simple experiments, Computer	Mathematics, physics, PP or JP
Sound Functions	15 years and older, Function, digital, sound, resolution	Experiments, Computer	Mathematics, music, technical, physics, PP or JP or TM
Parabola and Horizontal launch	15 – 16 years, Parabola, horizontal launch	Physical experiments	Mathematics, physics, PP or JP
Experiments to Investigate Decaying Processes	15 – 16 years (and older), (stunted) growth, decay, experiments, analysing measuring values, regression of measuring values, modelling	Experiments, Esp. reality related	Mathematics, Physics, PP or JP
Parabola and Technology	15 years and older, Parabola, car lights, Satellite dish	Experiments, Not-subject related topic	Mathematics, physics, technics, PP or JP
Logarithmic function	15 – 17 years, Function, logarithm, universe, sound, earthquake	Esp. reality related, Not-subject related topic	Mathematics, physics, geography PP or JP
Functional Relations 2	15 – 18 years, Concept of function, functional thinking, linear, square, anti-proportional and other functions, physical experiments in interdisci-	Physical experiments with long worksheets introduced by realis-	Mathematics, Physics, TM, PT, PP, or JP

	plinary lessons	tic situations	
Function x^(3/2) and x^(1/2): practical examples with pendulum	15 – 18 years, Function, potential function (rational powers), oscillation of pendulum	Physical experiment, Worksheets (spreadsheet)	Mathematics, Physics JP
Growth	15 – 18 years, Exponential and linear growth, functional relations, applications of growth, mathematical models, modelling	Worksheets, Esp. related to realistic contexts	Mathematics, Biology, geography, TM

Higher secondary level

Topic/ Theme	Age and Key-words	Special (interdisciplinary) approaches	Subjects involved and cooperation form
GPS and fair Insurance Premiums	15 – 19 years, Modelling, functional relations, reflecting	Worksheets, Esp. reality related	Mathematics, economy, TM or PP
X^-0,5, square root function in divisor	16 years old	Phenomena, experimental measurements with worksheets (spreadsheet)	Mathematics, physics, JP
Introduction of Trigonometric Functions	16 – 17 years, Trigonometric functions, circulation, oscillation	Experiments, Computer	Mathematics, physics, TM
Functional Relation 3	16 – 18 years, Concept of function, functional thinking, “unknown” functions in interdisciplinary contexts, experiments	Physical experiments with long worksheets, introduced by realistic situations	Mathematics, physics, JP (or PP)
Fermat´s Principle and Calculus	16 – 18 years, Optimization problems, calculus, Fermat’s principle	Physical experiments, introduced by realistic situation	Mathematics, physics, JP (or PP)
Paramecia	16 – 19 years, Logistic growth, modelling, functional relations, concept of derivative, rate of change	Worksheets, Computer, reality related	Mathematics, Biology, TM, PP
Modelling Things in Traffic	16 – 19 years, Modelling activities in the context of traffic	Worksheets, experiments at the street	Mathematics, physics (technics) PP, TM
Arc Length of a Plane Curve – “proved by physics”	18 – 20 years, Arc length, horizontal launch	Physical experiments	Mathematics, physics, PP or JP

4.2 Modules in different languages

The ScienceMath-modules are available in the following languages:
English, Slovenian, German, Finnish, Danish, French, Spanish, Turkish.
The languages concern the partner countries in the project as well as countries of applicants to the ScienceMath European teacher training event 2009 in Slovenia.
The Modules can be downloaded on the website www.sciencemath.ph-gmuend.de.
Some examples you´ll find in Annex 1.

4.3 All English Modules

All modules translated into English you´find on our website www.sciencemath.ph-gmuend.de <English flag or in Annex 2.

5 Dissemination and Sustainability

5.1 Website

The ScienceMath project offers a website: www.sciencemath.ph-gmuend.de.

Under the categories *Home, The Partners, Teaching material, Teacher training, Events, Evaluation, Literature; MACAS* you´ll find the teaching modules for download in different languages, presentations for download and many, many more material and information.

Figure 15: ScienceMath-Website, Cutting of a screenshot[6]

[6] 17th October 2009

5.2 Teacher Trainings

The main objective of the ScienceMath project was the development of tested teaching modules for the use in European classrooms. In 2.2 we describe Science-Math concepts for dissemination and especially for implementation in the class-room. A very important dissemination and implementation activity is teacher training resp. professional development PD. The developed ScienceMath concept allows short or long term versions. Core elements are the modules and prepared presentations. In Annex 1 and 2 resp. on our website you´ll find the developed modules and most of the presentations can be downloaded as well. Because of that comprehensive concept preparation the ScienceMath PD could be overtaken by other groups very easiliy, so that its realisation depends not only on the capacity of the projects´ members. In Slovenia 2009 we realised a trial run which was positive evaluated (3.1.4) and has to be seen as a good basis for sustainability.

5.3 Conference Reports and References

A central dissemination line of the ScienceMath project concern publication and presentation activities. The partners organised conferences, symposia and workshop at conferences, presented the material and published it in scientific and teacher journals. These activities took place very calculated with the aim of discussing the material and make it known to teachers, teacher educators and didactical researchers. The papers, proceedings and conference reports make the material everytime available and contributes to the sustainability of the project.

Very remarkable in connection to the ScienceMath project are the network conferences MACAS – *Mathematics and its connection to the Arts and Sciences*, organised of some of the project partners. For more information and the order address for the proceedings please see the website www.sciencemath.ph-gmuend.de > MACAS.

References of the published papers you´ll find on our website www.sciencemath.ph-gmuend.de > literature. The articles referred in this volume are summarised in the next chapter.

The following list informs about conference activities – period 2007 – 2009 as far as announced for this volume (Note: The project partner appears first on principle). Some of the mentioned presentations you`ll find to download on our website. For further information please visit the conference site or contact the respective project partner.

2007

26th – 30th March 2007; DMV –GDM conference Berlin, Germany
Claus Michelsen
Modellbildungsprozesse und Integration von Mathematik, Physik und Biologie
http://www.mathematik.uni-dortmund.de/ieem/new/index_f-bzmu.html

26th – 30th March 2007; DMV –GDM conference Berlin, Germany
Astrid Beckmann
ScienceMath – an interdisciplinary Euroepan project
http://www.mathematik.uni-dortmund.de/ieem/new/index_f-bzmu.html

26th – 30th March 2007; DMV –GDM conference Berlin, Germany
Astrid Beckmann

Fächerübergreifender Mathematikunterricht – Hintergrund, Argumente und mögliche Kooperationsformen – Eine Einführung zum gleichnamigen Minisymposium
http://www.mathematik.uni-dortmund.de/ieem/new/index_f-bzmu.html

26th – 30th March 2007; DMV –GDM conference Berlin, Germany
Thilo Höfer
Funktionales Denken fördern mit Hilfe von physikalischen Schülerexperimenten unter Einsatz von grafikfähigen Taschenrechnern
http://www.mathematik.uni-dortmund.de/ieem/new/index_f-bzmu.html

2007, Symposium conducted at 12th EARLI Biennial Conference for Research on Learning and Instruction, Budapest Hungary
Kaarina Merenluoto and Hurme, T-R., Salonen, P., & Järvelä, S.
Metacognition as a shared process in networked mathematical problem solving. In P. Alexander (Chair), Developing potentials for mathematics learning through metacognition.

29th – 31st May, 2007; 2nd International Symposium on Mathematics and its Connections to the Arts and Sciences, MACAS2, Odense, Denmark
Damjan Kobal
The Arithmetic Mean and Car Differential
ISBN: 978-87-92321-04-6

29th – 31st May, 2007; 2nd International Symposium on Mathematics and its Connections to the Arts and Sciences, MACAS2, Odense, Denmark
Tine Golež
Cooperation between mathematics and physics teaching – the case of horizontal launch.
ISBN: 978-87-92321-04-6

29th – 31st May, 2007; 2nd International Symposium on Mathematics and its Connections to the Arts and Sciences, MACAS2, Odense, Denmark
Marina Rugelj
Giving sense to math formula
ISBN: 978-87-92321-04-6

29th – 31st May, 2007; 2nd International Symposium on Mathematics and its Connections to the Arts and Sciences, MACAS2, Odense, Denmark
Astrid Beckmann
Mathematical literacy – through scientific themes and methods
ISBN: 978-87-92321-04-6

29th – 31st May, 2007; 2nd International Symposium on Mathematics and its Connections to the Arts and Sciences, MACAS2, Odense, Denmark
Claus Michelsen
Promoting students´interests in mathematics and science through interdisciplinary instruction
ISBN: 978-87-92321-04-6

29th – 31st May, 2007; 2nd International Symposium on Mathematics and its Connections to the Arts and Sciences, MACAS2, Odense, Denmark
Thilo Höfer
Fermat meets Pythagoras
ISBN: 978-87-92321-04-6

25th – 29th June, 2007; ICTMA 13 - International Conference on the Teaching of Mathematical Modelling and Applications, Kathmandu, Nepal
Marina Rugelj
Giving sense to the math formula

September 2007; Annual symposium of the Finnish mathematics and science education research association, University of Turku
Kaarina Merenluoto and Hurme, T-R
Examining shared metacognitive regulation in groups: the qualitative content analysis of the discussion forum data.

9th – 10th November, 2007; The 2007 Annual Meeting of the Society of Mathematicians, Physicists and Astronomers of Slovenia, Olimlje, Slovenija
Tine Golež
Moment and Inertia as an Inquiry Task

30th November – 1st December 2007; International Conference ComLab, Radovljica, Slovenia
Tine Golež
Real-time experiments approach in kinematics using ComLab equipment
ISBM: 978-961-253-009-9

2008

13th -18th March, 2008; 42. Jahrestagung der Gesellschaft für Didaktik der Mathematik, Budapest, Hungary
Damjan Kobal
Two Simple Math Ideas in Technology
http://www.mathematik.uni-dortmund.de/ieem/new/index_f-bzmu.html

13th -18th March, 2008; 42. Jahrestagung der Gesellschaft für Didaktik der Mathematik, Budapest, Hungary
Astrid Beckmann & Damjan Kobal & Claus Michelsen
The European ScienceMath Project
http://www.mathematik.uni-dortmund.de/ieem/new/index_f-bzmu.html

13th -18th March, 2008; 42. Jahrestagung der Gesellschaft für Didaktik der Mathematik, Budapest, Hungary
Claus Michelsen
Preparing the teachers for an interdisciplinary curriculum: Modelling courses for secondary education in-service teachers
http://www.mathematik.uni-dortmund.de/ieem/new/index_f-bzmu.html

13th -18th March, 2008; 42. Jahrestagung der Gesellschaft für Didaktik der Mathematik, Budapest, Hungary
Astrid Beckmann
Interdisciplinary lessons between Math and Art
http://www.mathematik.uni-dortmund.de/ieem/new/index_f-bzmu.html

13th -18th March, 2008; 42. Jahrestagung der Gesellschaft für Didaktik der Mathematik, Budapest, Hungary
Thilo Höfer
Einführung des Funktionsbegriffs in der Sekundarstufe I
http://www.mathematik.uni-dortmund.de/ieem/new/index_f-bzmu.html

13th -18th March, 2008; 42. Jahrestagung der Gesellschaft für Didaktik der Mathematik, Budapest, Hungary
Simon Zell
Erkunden des Variablenbegriffs durch physikalische Experimente
http://www.mathematik.uni-dortmund.de/ieem/new/index_f-bzmu.html

13th -18th April, 2008; 5th International Colloquium on the Didactics of Mathematics, Crete, Greece
Marina Rugelj
How to "domesticate" logarithms in school?

8th – 10th May 2008; Metacognition Sig Invited Symposium at the 3rd Earli Metacognition Sig –meeting, Ioannina, Greece
Kaarina Merenluoto and Hurme, T-R., Salonen, P. & Järvelä, S.
Socially Shared Metacognition in artimetic problem solving

6th -13th July 2008; 11th International Congress on Mathematics Education, Monterrey, Mexico
Damjan Kobal
The Arithmetic Mean and Car Differential

23th – 25th August 2008; 6th International Conference on Conceptual Change, Turku, Finland
Kaarina Merenluotot and Hurme T-R.
The role of motivational factors in mathematical problem solving demanding conceptual change.

28th August to 2nd September, 2008; 12th Serbian Mathematical Congress, Novi Sad, Serbia
Tine Golež
The use of motion sensor in the teaching of calculus.

25th – 27th October 2008; Journées APMEP La Rochelle, France
Tine Golež
LE CALCUL INFINITESIMAL ENTRE MATHEMATIQUES ET PHYSIQUES (PAR EXPERIENCES ET MESURES EN TEMPS REEL)
See: http://www.youtube.com/user/sciencemathproject

2008; ICLS International Conference for the Learning Sciences–Conference, Utrecht, Netherlands
Kaarina Merenluoto and Hurme, T-R., Salonen, P. & Järvelä, S.
How learners share and construct metacognition in social interaction.

2009

28th January – 1st February 2009, CERME, Lyon, France
Astrid Beckmann, Simon Zell, Jan Alexis Nielsen & Thilo Höfer
The European ScienceMath Project

28th January – 1st February 2009, CERME, Lyon, France
Simon Zell & Astrid Beckmann
Modelling activities while doing experiments to discover the concept of variable

2009; 13th Biennial Conferenc4e for Research on Learning and Instruction
Kaarina Merenluoto and Hurme, T-R. & Lehtinen, E.

Adults' reasoning about the density of numbers on the number line and its relation to their basic arithmetic skills

2009; 13th Biennial Conference for Research on Learning and Instruction
Kaarina Merenluoto and Hurme, T_R. & Järvelä, S.
A process oriented approach to examine joint regulation of group's problem solving – tracing socially shared metacognition.

2nd – 6th March 2009; Jahrestagung der GDM, Oldenburg, Germany
Astrid Beckmann
Fächerübergreifender Unterricht zwischen Mathematik und Biologie – Ernährungskreis, Ähnlichkeit und Allometrie
http://www.mathematik.uni-dortmund.de/ieem/new/index_f-bzmu.html

2nd – 6th March 2009; Jahrestagung der GDM, Oldenburg, Germany
Thilo Höfer
Funktionales Denken fördern
http://www.mathematik.uni-dortmund.de/ieem/new/index_f-bzmu.html

2nd – 6th March 2009; Jahrestagung der GDM, Oldenburg, Germany
Simon Zell
Mathematical Literacy
http://www.mathematik.uni-dortmund.de/ieem/new/index_f-bzmu.html

22nd – 24th March 2009; FISER Cyprus
Damjan Kobal
From mathematics to technology and backwards by intuition
ISBN: 978-975-8401-67-3 (p. 423 – 431)

22nd – 24th March 2009; FISER Cyprus
Marina Rugelj, Tine Golež
Bouncing ball – a mathematisation for second year high school and Matura students
ISBN: 978-975-8401-67-3 (p. 105 – 110)

22nd – 24th March 2009; FISER Cyprus
Tine Golež
The use of motion sensor in the teaching of mathematics
ISBN: 978-975-8401-67-3 (p. 405 – 414)

21th – 23th May 2009; MACAS 3, Moncton, Canada
Astrid Beckmann and Annika Grube
Cross-curricular Teaching between Mathematics and Biology – Nutrition Circle, Similarity and Allometry

21th – 23th May 2009; MACAS 3, Moncton, Canada
Claus Michelsen & Jan Alexis Nielsen
Interdisciplinarity through processes of Socio-mathematical Decision-making

21th – 23th May 2009; MACAS 3, Moncton, Canada
Simon Zell
Mathematical Literacy and how scientific experiments can promote that conception

21th – 23th May 2009; MACAS 3, Moncton, Canada
Astrid Beckmann

Advancing the Concept of Variable through cross-curricular stations between Arts and Mathematics instruction

24th – 28th June 2009; International History, Philosophy and Science Teaching Group Biennial Conference, University of Notre Dame, USA
Damjan Kobal
The Teacher's Authority and Responsibility – A Historical Perspective on Recent Pseudo-scientific Metrics in Education

14th – 15th July 2009; First International Geogebra Conference, Hagenberg, Austria
Damjan Kobal,
The use of GeoGebra to motivate, to present and to deepen the comprehension of mathematics, Keynote address

11th – 17th September, 2009; Models in Developing Mathematics Education, Dresden, Germany
Damjan Kobal
The use of technology to motivate, to present and to deepen the comprehension of math

11th – 17th September, 2009; Models in Developing Mathematics Education, Dresden, Germany
Tine Golež
Toward Calculus via Real-time Measurements

11th – 17th September, 2009; Models in Developing Mathematics Education, Dresden, Germany
Astrid Beckmann
Learning Mathematics through scientific contents and methods

11th – 17th September, 2009; Models in Developing Mathematics Education, Dresden, Germany
Simon Zell
Using physical experiments in mathematics lessons to introduce mathematical concepts

5.4 Following projects

Although the financial support of the ScienceMath project ended in 2009 the project continues. The material is used in teacher education and there are already planning activities for further teacher trainings and conferences. New projects have started. E.g. there is already a research line about the question of the meaning of interdisciplinary modules for different ethic groups/migrants (INSCIME project) etc. Last but not least the idea of joining a network is worked out.

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An overview about all the literature published by the ScienceMath partners you´ll find on the website www.sciencemath.ph-gmuend.de > literature.

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Annex 1

Modules in different languages - Examples

Annex 2

All English Modules